Subject: VCE Mathematical Methods

Unit 1

Week	Area of Study	Learning Focus
Headstart	Algebra	• Apply index laws to simplify expressions. • Understand manipulation of exponential expressions, including prime decomposition.
1 - 4	Functions and Graphs: Quadratic Functions, Factorisation, Solving equations.	• Employ expansion formulae eg: distributive law, 2 brackets, D.O.P.S., perfect squares • Become familiar with factorising formulae and to employ factorising techniques including with algebraic fractions, to simplify or solve equations: eg: common factor, substitution, distributive law, 2 brackets, D.O.P.S., perfect squares • Solve quadratic equations and inequations, using factorising, the Null Factor Law, Completing the Square and the Quadratic Formula • Determine the number and nature of real roots using the Discriminant • Identify key quadratic features: axis intercepts, turning point and axis of symmetry, maximum or minimum values • Find the Turning Point using the x-intercepts and Turning Point Form • Transformations of quadratic functions via dilations ('parallel to an axis' and 'from an axis'), reflections in an axis and translations, including using matrices • Sketch graphs of quadratic functions • Identify characteristics of families of quadratic functions • Solve simultaneous quadratic and linear equations • Apply quadratic functions to solve problems
5 - 6	Functions and Graphs: Hyperbolas, Truncus & Square Root Functions and Circles	• Recognise rules of the Rectangular Hyperbola, Truncus & Square Root Relations • Sketch graphs of the power relations $f(x) = x^n$, $n \in \{-2, -1, ½, 1/3\}$ • Apply transformations of the graphs to the form $y = a(x + b)^n + c$ • Identify key features of these 3 relations • Determine rules for these relations using specified information • Recognise the rule for basic circles the equation of a circle with a specified radius and centre • Identify key features of circles • Determine rules for circles using known information • Sketch graphs of circles • Apply translations to circles
7 - 10	Probability	• Employ terminology such as random experiments, sample spaces, outcomes, elementary and compound events • Understand principles of non-negative probabilities and the sum of probabilities is one. • Simulate experiments using simple random generators such as coins, dice, spinners and technology • Calculate experimental probabilities • Represent sample spaces and events: lists, grids, Venn diagrams, tree diagrams, Kanaugh tables (eg: conditional probabilities) • Interpret results, including informal consideration of proportions in samples • Apply Addition Rule: $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ • In multi-stage experiments, multiply probabilities when events occur together (AND) and add probabilities when alternative outcomes are possible (OR) • Apply Conditional Probability formula in terms of reduced sample space, the relations

		$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ or $\Pr(A \cap B) = \Pr(A \mid B) \times \Pr(B)$ • Determining Independence $\Pr(A).\Pr(B) = \Pr(A \cap B)$ • Determine Mutually Exclusive Events $\Pr(A \cap B) = 0$, hence $\Pr(A \cup B) = \Pr(A) + \Pr(B)$ • Apply probability concepts to worded problems • Understand the law of total probability for two events $\Pr(A) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B')\Pr(B')$ • Understand that for pairwise independent events A and B, $\Pr(A \mid B) = \Pr(A)$, $\Pr(B \mid A) = \Pr(B)$ and $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ • Informally consider sample proportions
11 - 13	Functions and Graphs: Notation, Domain and Range, Inverse functions, Modelling	• Become familiar with and employ set notation • Identify if a relation is a function and apply the Vertical Line Test • Identify types of correspondence • Employ notation for and determine the rule of a function, domain, (including maximal, natural or implied domain), co-domain and range of a relation. • Sketch graphs of functions including hybrid functions • Identify examples of relations that are not functions and their graphs such as $x = k$, $x = ay^2$ and circles • Become familiar with the relationship between the graph of a one-to-one function, its inverse function and reflection in the line $y = x$ • Determine the inverse of a one –to-one function • Employ equations to solve modelled real world problems • Apply transformations: Dilations, Reflections, Translations
14 - 17	Functions and Graphs: Cubic and quartic functions.	• Add, subtract, multiply and divide polynomials (polynomial long division) • Apply formulae for expanding perfect cubes • Factorise cubes using polynomial long division or the sum or difference of cubes formulae • Employ the Remainder and Factor Theorems and Rational Root Theorems to identify linear factors of cubics and quartics • Solve cubic or quartic equations and inequations numerically (including Numerical Approximation of Roots using the Bisection Method), graphically and algebraically • Sketch graphs of functions to degree 4 and other polynomials of higher degree such as $g(x) = (x + 2)^2 (x - 1)^3 + 10$ • Determine the rules for given cubic graphs • Understand connections between the roots of a polynomial function, its factors and the horizontal axis intercepts of its graph • Recognise the key features and properties of power and polynomial functions and their graphs • Apply transformations of the plane, dilation, reflection in axes, translation and simple combinations of these transformations • Employ cubic equations to solve modelled real world problems
18 - 20	Algebra Functions and Graphs: Matrix notation and applications. Using matrices for transformations. Applying transformations to sketch graphs.	• Become familiar with notation for transformations • Simplify using addition, subtraction and scalar multiplication • Interpret and apply matrices to represent points and transformations • Employ matrix equations to determine curve images under linear transformations

Unit 2

Week	Area of Study	Learning Focus
1 – 2	Algebra Functions and Graphs: Logarithms	• Convert between index and log statements • Simplify using $\log_{10}$, $\log_e$ and other bases • Evaluate logarithmic expressions • Distinguish between exact values and approximate values • Simplify and solve equations using the 5 log laws and other logarithmic methods: eg $a^{\log_a x} = x$ and $\log_a(a^x) = x$ • Recognise the 'change of base' law • Sketch functions of the form $f : R+ \rightarrow R$, $f(x) = \log_a(x)$, where $a > 1$, and the inverse function of $y = a^x$, • Apply basic graphical transformations of $y = \log_a(x)$ • Apply logarithmic scales
3 - 4	Probability	• Introduce Addition and Multiplication principles • Define and apply the concept of arrangements (ie Permutations) • Define and apply the concept of selections (ie Combinations) • Apply counting methods to probability • Relate combinations to Pascals triangle and the binomial theorem.
5 - 7	Algebra Functions and Graphs: Circular Functions Trigonometric equations	• Review Basic Trigonometry and the relationship of $\tan(x) = \sin(x)/\cos(x)$ • Understand unit circle definitions • Determine the exact value of trigonometric special angles • Convert between radians and degrees and determine arc length • Employ CAST, symmetry rules and complementary relations • Solve trigonometric equations including those of worded problems • Determine the amplitude, period or mean value of circular functions • Sketch graphs of circular functions • Apply transformations of sine and cosine graphs; • Employ the identity $\sin^2(x) + \cos^2(x) = 1$ • Calculate $\sin(x) \approx x$ for small values of x • Use inverse functions and transformations to solve equations of the form $Af(bx) + c = k$
8 - 10	Calculus Functions and Graphs Algebra: Rates of Change Velocity	• Use graphical, numerical and algebraic approaches to estimate or find an exact value for the gradient of a secant or tangent to a curve at a given point • Calculate gradient of a secant or average rates of change • Calculate rates of change of Polynomials and Hybrid Functions • Estimate instantaneous rates of change • Use the gradient of a tangent at a point on a graph to describe and measure instantaneous rate of change • Identify rates of change: constant, variable, average and zero • Interpret graphs with respect to rate of change such as temperature or pollution levels over time, and the height of water in containers of different shapes that are being filled at a constant rate • Relate velocity-time graphs to position-time graphs • Consider rates of change in a variety of practical contexts • Informally consider continuity and smoothness and instantaneous rate of change as a limiting case of the average rate of change
11 - 13	Calculus Functions and Graphs Algebra Differentiation	• Employ notations for the derivative of a function: $f'(x)$, dy/dx, $d/dx\,(f(x))$, $Dx(f)$ • Differentiate by First Principles • Differentiate by rule including power functions and simple polynomial functions • Become familiar with increasing and decreasing functions and gradient sign diagrams

		• Determine the gradient of a tangent to a curve at a point, via the derivative • Employ differentiation to solve problems • Become familiar with the concept of a limit • Evaluate limits including for hybrid functions and using CAS technology Determine continuity and differentiability
14 - 18	Calculus Functions and Graphs Algebra: Applications of Differentiation and Antidifferentiation	• Determine the equation of the tangent and normal at a given point on a curve. • Employ the derivative in rate of change problems • Apply derivatives to locating stationary points and determining their nature: maxima, minima and point of inflection • Sketch graphs employing Calculus • Apply derivatives to solving maximum or minimum problems • Apply derivatives to kinematics • Find a family of derivative functions for a given power or polynomial function • Determine the numerical approximation of roots of cubic polynomial functions using Newton's method. • Employ notation to determine an antiderivative • Antidifferentiate by rule including power functions • Applying antidifferentiation to rates of change or kinematics problems involving straight line motion • Find a family of derivative functions for a given power or polynomial function • Determine a specific antiderivative given a boundary condition